**Screening Test**

**Second Round**

Time: 2:15 minutes

Q1.[10 Marks]\* An evil king has 1000 bottles of medicine. A neighboring queen plots to kill the bad king, and sends a servant to poison the medicine. The king's guards catch the servant after he has only poisoned one bottle. The guards don't know which bottle was poisoned, but they do know that the poison is so potent that even if it was diluted 1,000,000 times, it would still be fatal. Furthermore, the effects of the poison take one month to surface. The king decides he will get some of his prisoners in his vast dungeons to drink the medicine.

The king has only 40 prisoners in his prison house to figure out which medicine has poison. He has also has only 50 days to figure out using these prisoners otherwise the king would die because of not getting medicine.

Help the king to do this task.

Q2. (a) [5 Marks]\* Egg problem: You have two eggs. You need to figure out how high an egg can fall from a 225 story building before it breaks. The eggs might break from the first floor, or might even survive a drop from the 225th floor -- you have no a priori information. The largest number of egg drops you would ever have to do to find the right floor is of O(n1/2). Explain how.

(b) [5 marks]\* After you've solved the above problem, generalize. Define the "break floor" as the lowest floor in a building from which an egg would break if dropped. given an n story building and a supply of d eggs, find the strategy which minimizes (in the worst case) the number of experimental drops required to determine the break floor. What is the order of your algorithm?

Q3. [10 Marks] \*10 straight-jacketed prisoners are on death row. Tomorrow they will be arranged in single file, all facing one direction. The guy in the front of the line (he can't see anything in front of him) will be called the 1st guy, and the guy in the back of the line (he can see the heads of the other nine people) will be called the 10th guy. The prisoners can hear what their neighboring prisoners mutter.

An executioner will then put a hat on everyone's head; the hat will either be black or white, totally random. Prisoners cannot see the color of their own hat. The executioner then goes to the 10th guy and asks him what color hat he is wearing; the prisoner can respond with either "black" or "white". If what he says matches the color of the hat he's wearing, he will live. Else, he dies. The executioner then proceeds to the 9th guy, and asks the same question, then asks the 8th guy ... this continues until all of the prisoners have been queried.

This is the night before the execution. The prisoners are allowed to get together to discuss a plan for maximizing the number of lives saved tomorrow. What is the optimal plan?

Q4. [10 marks]\* Find elements A[i] and A[j] in a sorted array such that |A[i]| + |A[j]| == k i.e. abs(A[i]) + abs(A[j]) == k. It is obvious that array can have negative elements

Q5. [10 Marks]\* Mohan and Laksmi are staying in different rooms in the same hotel. Mohan needs to give a gold pendant to Laksmi, but spies are trying to assassinate Mohan and Laksmi so neither of them can leave their room. The only way they can transfer objects is by using the bellhops. Both Mohan and Laksmi have a safe with a large clasp that can be secured with a padlock. Both Mohan and Laksmi have a padlock and a corresponding key. (So 1 gold pendant, 2 safes, 2 padlocks, and 2 keys.) But the bellhops are thieves. Anything that is not padlocked in the safe will be stolen by the bellhops - including any unlocked padlocks, the keys or the pendant. How can Mohan transfer the gold pendant to Laksmi without it being stolen?

Q6. (a) [5 Marks] Find median of a Binary Search Tree using any method and any time/space complexity.

(b) [5 Marks] Now find it in one traversal and use constant space. Note that the tree could be unbalanced.

Q7. (a) [5 Marks] Write Solution to integer subset problem. That is there is a set of integers and you have to find if there is a subset of integers that would sum to a given number k. Only explain the method.

(b) [5 Marks]\* Now, the constraint that an integer can occur only once is relaxed. That is an integer can occur multiple times. Now write solution to modified integer subset problem.

Q8. [10 Marks] \*Four people, A, B, C, and D, are on one side of a bridge, and they all want to cross the bridge. However, it's late at night, so you can't cross without a flashlight. They only have one flashlight. Also, the bridge is only strong enough to support the weight of two people at once. The four people all walk at different speeds: A takes 1 minute to cross the bridge, B takes 2 minutes, C takes 5 minutes, and D takes 10 minutes. When two people cross together, sharing the flashlight, they walk at the slower person's rate. How quickly can the four cross the bridge?

Q9. [10 Marks]\* There are 3 black hats and 2 white hats in a box. Three men (we will call them A, B, & C) each reach into the box and place one of the hats on his own head. They cannot see what color hat they have chosen. The men are situated in a way that A can see the hats on B & C's heads, B can only see the hat on C's head and C cannot see any hats. When A is asked if he knows the color of the hat he is wearing, he says no. When B is asked if he knows the color of the hat he is wearing he says no. When C is asked if he knows the color of the hat he is wearing he says yes and he is correct. What color hat and how can this be?

Q10 (a) [5 Marks]\* There are three closed and opaque cardboard boxes. One is labeled "APPLES", another is labeled "ORANGES", and the last is labeled "APPLES AND ORANGES". You know that the labels are currently misarranged, such that no box is correctly labeled. You would like to correctly rearrange these labels. To accomplish this, you may draw only one fruit from one of the boxes. Which box do you choose, and how do you then proceed to rearrange the labels?

(b) [5 Marks]\* You are an archaeologist that has just unearthed a long-sought pair of ancient treasure chests. One chest is plated with silver, and the other is plated with gold. According to legend, one of the two chests is filled with great treasure, whereas the other chest houses a man-eating python that can rip your head off. Faced with a dilemma, you then notice that there are inscriptions on the chests:

Silver Chest Gold Chest

This chest contains the python. One of these two inscriptions is true.

Based on these inscriptions, which chest should you open?